STUDIES IN THEORETICAL PHILOSOPHY

Herausgegeben von Tobias Rosefeldt
und Benjamin Schnieder

in Zusammenarbeit mit

vol. 3

VITTORIO KLOSTERMANN

KATHARINA FELKA

Talking About Numbers

Easy Arguments for Mathematical Realism

VITTORIO KLOSTERMANN

Bibliographische Information der Deutschen Nationalbibliothek
Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliographie; detaillierte bibliographische Daten sind im Internet über *http://dnb.dnb.de* abrufbar.

Gedruckt auf Alster Werkdruck der Firma Geese, Hamburg,
alterungsbeständig ISO 9706 und PEFC-zertifiziert.
Druck: docupoint GmbH, Magdeburg
Printed in Germany
ISSN 2199-5214
ISBN 978-3-465-03879-5

By a gentle river laid,
Thirsis to his Phillis said,
'Equal to these sandy grains,
Is the number of my pains [...]'

Sir Edward Sherburne

Acknowledgments

This book is based on my PhD thesis that I submitted in October 2014 at the University of Hamburg. My thesis was supervised by Benjamin Schnieder and examined by Tobias Rosefeldt and Stephen Yablo. I am extremely grateful for many valuable comments and suggestions they made on the material contained in the book. I would especially like to thank Tobias for supporting me in various ways since the very beginning of my academic career and for infecting me with his enthusiasm about theoretical philosophy in general and the topic of my thesis in particular.

I wrote my PhD thesis in the Nominalizations project at the University of Hamburg. Other members of the project were Stephan Krämer, Robert Schwartzkopff, and Alex Steinberg. Many thanks to them for providing an inspiring atmosphere and fruitful discussions. Let me also thank Isa Kerem Bayirli, Ralf Busse, Catharine Diehl, Daniel Dohrn, Peter Fritz, Thomas Hofweber, Maria Kuper, Lisa Marani, Michael Potter, Bernhard Salow, Moritz Schulz, Barbara Vetter, and Julia Zakkou either for very helpful discussions or for being around as friends and supporting me morally during that time (or both). I am particularly grateful to Alex who made extremely helpful comments and suggestions regarding both content and form of the thesis, for being there, and for putting up for months with a non-existent king.

My PhD thesis received the Wolfgang-Stegmüller award in 2015. Many thanks to Mrs. Margret Stegmüller for her generosity as well as to the committee involved. I would also like to thank the German Research Foundation (DFG) and the French National Research Agency (ANR) that funded the Nominalizations project as well as the German Academic Exchange Service (DAAD) for funding a research stay at the Massachusetts Institute of Technology that allowed me to spend nine extremely fruitful months working with Stephen Yablo.

Finally, I would like to thank my family for their continuing support.

Summary

This book is concerned with easy arguments for the existence of numbers. A paradigmatic easy argument starts with the uncontroversial premise that the sentence ‘Mars has two moons’ is true and proceeds as follows: If the sentence ‘Mars has two moons’ is true, then the sentence ‘The number of moons of Mars is two’ is true as well. The truth of the latter sentence demands of the world that it contain numbers and, thus, numbers exist. The existence of such easy arguments is surprising, since the question of whether numbers exist seems to be a hard question that does not have a trivial affirmative answer. The book, thus, investigates whether the premises of such arguments are correct. The first part examines the cogency of the first premise. There are two principled ways to deny it: fictionalism and indifferentialism. However, it is argued that neither of the two have an adequate explanation for the fact that the sentences ‘Mars has two moons’ and ‘The number of moons of Mars is two’ are not felt to differ in truth-value although they allegedly do. In the second part, the second premise is scrutinized. It is argued that this premise is based on a widely accepted but false linguistic analysis of the relevant number sentences that was proposed by Gottlob Frege. After a prominent alternative analysis is rejected as well, an account of such sentences as disguised question-answer pairs is defended. On this basis, it is argued that we should reject the second premise of easy arguments for the existence of numbers, since its justification is based on a false assumption about the linguistic structure of the relevant number sentences. Hence, the book concludes, the existence of numbers is not as easily provable as the easy arguments would have us believe.

Contents

Chapter 1

Introduction

This book is concerned with a certain puzzle that arises when we consider ontological disputes about the existence of numbers. It is constituted by two observations. When philosophers discuss the question of whether numbers exist, they usually assume that this is a very hard question that does not have an easy answer—a view which is supported by the fact that the question has been discussed for more than a thousands years and still has not been settled yet.[1] However, on the other hand, there seem to be very easy arguments for the existence of numbers. Such arguments start from a commonly accepted assumption and derive from it the existence of numbers with only two apparently uncontroversial premises. But if the existence of numbers can be derived as easily as such arguments make us believe, then the question arises of whether ontological disputes about the existence of numbers are fundamentally mistaken.

Let us consider such easy arguments for the existence of numbers in more detail. A paradigmatic easy argument starts with an ontologically innocent assumption that is commonly accepted as true. For instance, it is commonly accepted that Mars has two moons and that therefore sentence (1) is true:[2]

(1) Mars has two moons.[3]

This sentence is ontologically innocent since it only says something about concrete things, namely Mars and moons. Accordingly, its truth requires the existence of Mars and moons but nothing in addition. However, if sentence (1) is true, then sentence (2) is true as well:

(2) The number of moons of Mars is two.

This sentence is ontologically loaded since it says something about a number and, thus, its truth requires the existence of numbers.[4] However, if sentence

[1] Relevant discussion can already be found in Plato's *Sophist* (cp., e.g., Duke et al. (1995)). In the more recent literature this question is discussed in Benacerraf (1973), Hale (1988), Hale and Wright (2001), and Künne (2007), among others.

[2] I take the assumption that Mars has two moons to be the uncontroversial assumption but tacitly assume in the following that anybody who makes this assumption should also accept that sentence (1) is true.

[3] In the following I regard sentence (1) as short for 'Mars has exactly two moons'.

[4] Cp. Hofweber (2007: 2f.) for the terminological distinction between *ontologically innocent* and *ontologically loaded* sentences.

(2) is true and its truth requires the existence of numbers, then numbers exist! Hence, the existence of numbers seems to follow immediately from the commonly accepted assumption that sentence (1) is true.

This book examines easy arguments of the form just outlined, i.e. arguments that start from an ontologically innocent sentence of the form 'a has n Fs' and proceed to a seemingly ontologically loaded sentence of the form 'The number of Fs of a is n' (where 'n' is substituted with a number word).[5] Alternative easy arguments of this form are e.g. arguments that start from the sentences 'The Pyramid of Cheops has four corners', 'Obama has two hands', or 'Germany has sixteen federal ministers' and proceed to the sentences 'The number of corners of the Pyramid of Cheops is four', 'The number of Obama's hands is two', or 'The number of federal ministers of Germany is sixteen'. Such easy arguments trace back to Gottlob Frege who noted the close relationship between the sentences of the respective pairs and claimed that sentences of the form 'The number of Fs of a is n' are ontologically loaded.[6] However, the debate about easy arguments only really took up speed with the recent focus on meta-ontological questions.[7] For, as pointed out above, easy arguments raise serious doubts about the reasonability of ontological disputes about the existence of numbers.

How can we react to the observation that there are easy arguments for the existence of numbers? Of course, we might simply say that such arguments show that disputes about the existence of numbers are pointless and, thus, that philosophers should focus on other questions. However, some philosophers have tried to show that such a reaction would be premature. The most prominent strategy to defend ontological disputes about numbers in the recent literature derives from Kit Fine's work on realism. Fine accepts that the existence of numbers can be proven quite easily. However, according to Fine, ontological disputes about numbers have to be understood differently than outlined: they do not concern the question of whether numbers exist but rather the question of whether numbers exist *in reality*.[8] While the answer to the first question is trivial, the answer to the second question is not trivial at all. This strategy has many adherents in recent literature albeit it is often spelled out in different ways. For instance, some philosophers argue that ontological disputes about numbers do not concern the question of whether numbers exist but rather the question of whether numbers exist on a fundamental level of reality or, alternatively, whether numbers EXIST—while the predicate 'EXISTS' captures the most natural meaning of the existence predicate.[9]

[5] When I talk in the following about easy arguments for the existence of numbers without further qualification, I always mean easy arguments for the existence of numbers that have this form.

[6] Cp. Frege (1884: §57).

[7] Cp. Chalmers et al. (2009) for discussions of meta-ontological questions. For instance, Dorr (2008), Hofweber (2005a), Rayo (2013), Schiffer (2003), and Yablo (2000) are specifically concerned with easy arguments for the existence of numbers. The historically most influential criticism of the view that ontological questions are hard questions is due to Carnap (1950).

[8] Cp. Fine (2001).

[9] The first proposal is most prominently defended by Schaffer (2009), the second one by Sider (2009).

This strategy is confronted with the challenge of spelling out in more detail what exactly the hard metaphysical question is that philosophers have allegedly been interested in all along when seemingly discussing existence questions. This is no easy task, and none that will be taken up in this book. Rather, I argue that we should part company with the ontological deflationist even earlier than the Finean realist is willing to, and critically reexamine the alleged easy arguments rather than accepting their conclusion. Once we do so, it turns out that, despite initial appearances, they are not convincing, and, thus, do not threaten the legitimacy of ontological disputes about the existence of numbers. In a nutshell, here is why the easy arguments do not suceed: apart from the commonly accepted assumption that sentence (1) is true, the paradigmatic easy argument considered above relies on the following two premises:

(i) If sentence (1) is true, then sentence (2) is true.

(ii) The truth of sentence (2) demands of the world that it contain numbers.

The second premise was most famously defended by Gottlob Frege. According to Frege, sentence (2) is an identity sentence in which the expression 'is' is flanked by the two singular terms 'the number of moons of Mars' and 'two' that have the semantic function of referring to a number.[10] Since sentences that contain such singular terms are true only if they succeed in referring to a number, Frege claimed that the truth of sentence (2) requires the existence of a number. However, in this book I argue with the aid of certain linguistic observations that Frege's analysis of sentence (2) is mistaken: contrary to what Frege assumed, sentence (2) is not an identity sentence. Furthermore, the correct linguistic analysis of sentence (2) uncovers that its truth does not require the existence of numbers and, thus, that the second premise of the argument is false.

However, the denial of the second premise of the easy argument considered above is not the only option to criticize such arguments. Some philosophers think that rather the first premise of the argument is false. In the first part of the book I therefore investigate whether we can reject the first premise by maintaining that only sentence (1) but not sentence (2) is true. If we deny the first premise, then we are confronted with the challenge of explaining why the sentences (1) and (2) seem obviously truth-conditionally equivalent, although only sentence (1) but not sentence (2) is true. I discuss two accounts to meet this challenge that I take to be the most important ones, since they are both well-elaborated and based on a particularly promising idea: fictionalism and indifferentialism. These accounts trace back the seeming truth-conditional equivalence of the sentences (1) and (2) to what speakers assert in uttering the sentences. The accounts differ very much in detail but their common idea is that what speakers assert in uttering sentence (2) is true if what speakers assert

10 In contrast, Frege says, in the sentence 'Mars has two moons' the number word 'two' functions similarly to an adjective like 'green' in the sentence 'The tree has green leaves' (cp. Frege (1884: §57)). Later we will see that this is not quite correct but something in the vicinity.

in uttering sentence (1) is true (and vice versa). Therefore, it seems to us that sentence (2) is true if sentence (1) is true although in reality only what speakers assert in uttering sentence (2) is true if what speakers assert in uttering sentence (1) is true. But even though the general idea of these accounts appears initially attractive, we will see that both are ultimately unconvincing. This renders a denial of the first premise unattractive: without a plausible explanation of why sentences (1) and (2) seem obviously truth-conditionally equivalent although they are not, simply denying this premise is dialectically ineffective.

In the second part of the book I then investigate the second premise of the easy argument for the existence of numbers. As pointed out, this premise is based on a certain linguistic analysis of sentence (2) that goes back at least to Frege. According to this analysis, sentence (2) is an identity sentence that contains singular terms that have the semantic function of picking out a number. To evaluate the second premise, I investigate whether this analysis is correct. First, I discuss an alternative analysis of sentence (2) that has been developed in recent literature, which is based on some intriguing linguistic observations. According to this analysis, sentence (2) is not an identity sentence but rather has the same underlying structure as sentence (1). Accordingly, sentence (2) is ontologically innocent, too, in that its truth does not demand of the world that it contain numbers. However, I argue that the relevant linguistic observations do not support this analysis but rather must be explained in a different way. Therefore, this proposal cannot establish that sentence (2) is ontologically innocent after all. Second, I develop an alternative analysis of sentence (2) according to which this sentence is not an identity sentence. As I argue, the relevant linguistic observations as well as further linguistic data show that sentence (2) is a disguised question-answer pair. Due to the specific structure of the disguised question-answer pair, this analysis has the consequence that the truth of sentence (2) does not require the existence of numbers. Accordingly, I conclude, the argument is based on a false premise.

In the final chapter I briefly discuss the implications of this result. First, there are other allegedly easy arguments for the existence of numbers. However, closer examination will show that these arguments are actually not easy at all. Second, there are also easy arguments for the existence of other abstract objects, for instance for the existence of properties or propositions. We will see that some results of this book are specific to easy arguments for the existence of numbers, while others can be generalized.

Finally, let me make two clarifications concerning the scope of the question dealt with in this book. First, I only want to show that *easy* arguments for the existence of numbers are not convincing, i.e. that the existence of numbers cannot be derived from an ontologically innocent assumption that is commonly accepted as true. However, I do not presume to be able to show that there are no *other* convincing arguments for the existence of numbers. Accordingly, I do not aim to show that numbers do not exist, but only that it is not a trivial matter to settle the question. In this respect, it should be noted that the easy arguments targeted here are a special case of a more general argumentative move that might be called *argument from abstract reference*.[11] An argument of the general type tries to establish the existence of numbers on the basis of the observation that there are true number sentences whose truth requires the existence of numbers. However, there are many such arguments that do not qualify as easy arguments in the sense intended here. For instance, very often proponents appeal to the (alleged) truth of sentences from pure mathematics such as '2 and 2 are 4' or 'The number 2 is prime'. But whereas the assumptions of easy arguments are hardly debatable and seemingly ontologically innocent claims such as that Mars has two moons, alleged truths of pure mathematics are hardly ontologically innocent at all. Thus, an opponent might simply defuse the not so easy arguments from abstract reference by denying their premises. Since easy arguments, thus, promise to be dialectically more effective, I focus on this species of the more general kind.

Second, it should be pointed out that this book also has a far more general, methodological result concerning the question of how we should proceed to examine easy arguments. As becomes obvious in the course of the following discussion, we have to carefully investigate the linguistic assumptions on which such arguments are based. In order to do so, we have to take into account various linguistic data, for instance data concerning the information structure or the syntax of the sentences in question. In this task, it is absolutely crucial that philosophy interacts with linguistics, whose core business consists, after all, in the discovery and analysis of the relevant data. But up to now there has only been little interaction. The chapters to come show that this is a serious shortcoming in the philosophical literature: it transpires that linguistic data on syntax and information structure have an immediate bearing on ontological questions. Rather than ignoring them, philosophers should take into account and contribute to results from linguistic theory on these phenomena. This book should be read as a sustained defense for and a contribution to this project.

[11] Cp., e.g., Hale (1988), Künne (2007), and Loux (1978).

Chapter 2

Preliminaries

Let me start with some preliminary clarifications. As presented, sentence (1) is ontologically innocent, while sentence (2) seems ontologically loaded:

(1) Mars has two moons.

(2) The number of moons of Mars is two.

Sentence (1) is ontologically innocent, for its truth demands of the world that it contain concrete things like Mars and moons but nothing in addition. In contrast, sentence (2) is ontologically loaded, since its truth demands of the world that it contain a number in addition to Mars and moons. This difference is due to the fact that sentence (2) contains the number words 'the number of moons of Mars' and 'two' that have the semantic function of picking out a number. Like sentence (2), sentence (1) also contains a number word, namely 'two'. But in sentence (1) it does not have the semantic function of picking out a number. Hence, what demands the truth of a sentence imposes on the world depends on the semantic function of the expressions contained in it. In the following I give a short introduction to type theory, since type theory allows us to specify the semantic function of expressions more precisely. After introducing type theory, I consider the semantic function of the number words contained in the sentences (1) and (2) in more detail. Finally, I introduce a distinction that plays an important role in the following discussion, namely one between the demands the truth of a sentence imposes on the world and the demands the correctness of utterances of the sentence imposes on the world.

2.1 Type Theory

In general, the truth-value of a sentence is determined by the meaning of the sentence. Take, for instance, the sentence 'Kilpkonnad ela kaua'. If this sentence means that turtles live long, then the sentence is true. But if the sentence means that mayflies live long, then the sentence is false. The meaning of a sentence is partly determined by the meaning of its parts. For instance, it depends on the meaning of the expressions 'kilpkonnad', 'ela', and 'kaua' whether the sentence means that turtles live long or whether it does not. However, the meaning of a sentence is not only determined by the meaning of its parts but also by their syntactic configuration. Take, for instance, the sentences ‘John loves Paula’ and ‘Paula loves John’. These sentences differ in their meaning, although they contain the same expressions ‘John’, ‘loves’, and ‘Paula’. This difference in meaning is due to the fact that the expressions are differently arranged. Type theory explains how the meaning of a sentence results from the meaning of its parts and their syntactic configuration. The basic idea of type theory is that the meaning of a sentence results from the meaning of its parts via a process of functional application that is dictated by the syntactic structure of the sentence. Let me start with some preliminary remarks on (i) meaning, (ii) syntactic structure and (iii) functional application. After these remarks, we are in a position to consider the basic idea of type theory in more detail.

2.1.1 Meaning

As presented, the truth-value of a sentence is determined by its meaning and the meaning of a sentence is partly determined by the meanings of its parts. But what exactly are meanings? This is a very hard question that goes far beyond the scope of the present book. However, it is still important to draw a distinction between two components of the meaning of expressions, namely their *extensions* and their *intensions*. The extensions of expressions are also called their *semantic values* or *denotations*. Take, for instance, the sentence ‘Paula smokes’. The extension of the name ‘Paula’ is the person Paula and the extension of the predicate ‘smokes’ is the set of entities that smoke. The extension of the complete sentence is a truth-value. If we identify the meaning of expressions with their extensions, then we can account for the difference in meaning of the names ‘Paula’ and ‘John’ or the predicates ‘old’ and ‘young’, since these expressions have different extensions. However, extensions are still not sufficient to capture the meaning of expressions. To illustrate this, suppose that all smokers in our world are drinkers and vice versa. Then the predicates ‘smokes’ and ‘drinks’ have the same extension. But the predicates still differ in their meaning. Further, suppose that Paula smokes and that John smokes. Then the sentences ‘Paula smokes’ and ‘John smokes’ have the same extensions. But the sentences still differ in their meaning. *Intensions* are a more attractive candidate to capture the meaning of expressions. They are generalizations of extensions, namely extensions across possible worlds.[1] For instance, the intension of the predicate ‘smokes’ is a set of pairs where each pair consists in the world of evaluation and the set of entities that smoke in this world. Similarly, the intension of the predicate ‘drinks’ is a set of pairs where each pair consists in the world of evaluation and the set of entities that drink in this world. The intensions of the predicates ‘smokes’ and ‘drinks’ certainly differ, since the extensions of ‘smokes’ and ‘drinks’ differ in other possible worlds even if they do not differ in our world. Thus, intensions can account for the difference in meaning of the predicates ‘smokes’ and ‘drinks’. However, although intensions are the more attractive candidate to capture the meaning of expressions, I focus

[1] Cp. Forbes (2006: 9).

in this section only on their extensions. Accordingly, if I speak of the meaning of an expression, then I mean its extension. This focus on the extensions of expressions allows us to present the basic idea of type theory in a simpler way.

2.1.2 Syntactic Structure

As pointed out, the meaning of a sentence does not only depend on its parts but also on their syntactic configuration. In general, we can understand the syntactic structure of a sentence as the input of its semantic interpretation. To interpret a sentence semantically, we therefore need to understand the syntactic structure of the sentence first. In the following I distinguish two levels of syntactic structure: *surface structure* and *logical form*. The surface structure of a sentence is the syntactic structure that we can see when we look at the sentence, while the logical form of a sentence is the syntactic structure that enters its semantic interpretation. These two levels of syntactic structure have to be distinguished, since the logical form of a sentence can come apart from its surface structure. This happens in cases of ellipsis, for instance. Take, for illustration, the sentence 'John came to the party and Paula as well'. In the second part of the sentence the verb phrase 'came to the party' is deleted. But although the verb phrase is deleted on the surface it is still present at the level of logical form and thus enters the semantic interpretation of the sentence. Accordingly, the second part of the sentence is semantically interpreted in the same vein as the sentence 'Paula came to the party as well'.[2] Note that the linguistic notion of logical form that is relevant here is different from the philosophical notion of logical form.[3] Following Jason Stanley, we can call the linguistic notion of logical form a *descriptive* notion. According to the linguistic notion, the logical form of a sentence is its real syntactic structure that can be detected by empirical investigation. In contrast, the notion of logical form that is widespread in the philosophical literature is a *revisionary* notion. According to this notion, natural language is not precise enough for the purpose of scientific investigation. For this reason one might replace expressions from natural language 'by a notation which explicitly reveals the hidden contribution of logical expressions, such as the language of the predicate calculus.'[4] To give an example, Russell claims that the logical form of a sentence like 'The discoverer of Australia is famous' is represented by the sentence '$\exists! x Dx \wedge \forall x(Dx \rightarrow Fx)$'.[5] But with this claim Russell does not want to say that the real syntactic structure of the former sentence is represented by the latter sentence. Rather, he only wants to claim that the latter sentence makes the hidden contribution of logical expressions explicit, in contrast to the former sentence. However, later we will consider examples in which the logical form of a sentence in the linguistic sense is rather similar to the logical form of a sentence in the philosophical sense. This raises the question of whether the linguistic and the philosophical notion

2 Cp. Heim and Kratzer (1998: 248ff.) for further details.
3 Cp. Harman (1970) for the linguistic notion of logical form and Sainsbury (2001: 339ff.) for the philosophical notion.
4 Cp. Stanley (2000: 392).
5 Cp. Russell (1905).

of logical form are more closely connected than often assumed—a question that is left open in the following.

2.1.3 Functional Application

As pointed out, the meaning of a sentence results from the meanings of its parts via a process of functional application. Functional application is the application of *functions* to *arguments*. The application of functions to arguments yields *values*. To take an example from mathematics, the expression '$()^2$' denotes a function. This function takes numbers as arguments and yields numbers as values. If we take 2 as argument, then the function yields 4 as value, if we take 3 as argument, then the function yields 9 as value and so on. There are also mathematical functions that have truth-values as values. For instance, the function denoted by the expression '$() + 3 = 5$' yields the truth-value True if we take 2 as argument and the truth-value False if we take 3 as argument. As previously presented, the semantic value of a sentence like 'Paula smokes' is a truth-value as well. Gottlob Frege famously discovered that the semantic value of such a sentence is the output of a process of functional application as well. According to Frege, the semantic values of the expressions contained in a sentence have either the role of arguments or the role of functions that apply to arguments. In the case of the sentence 'Paula smokes', the semantic value of the expression 'Paula' has the role of an argument and the semantic value of the expression 'smokes' has the role of a function that is applied to this argument. More particularly, the semantic value of 'smokes' is a function that maps any entity x to the truth-value True if x smokes and to the truth-value False if x does not smoke. If we apply this function to Paula, then it yields the truth-value True if Paula smokes and the truth-value False if Paula does not smoke. In this way the semantic value of the sentence 'Paula smokes' results from the semantic values of the expressions contained in it via a process of functional application. Frege's discovery that the semantic value of a sentence is the output of a process of functional application laid the groundwork for modern type theory that we can consider after one more clarification.

2.1.4 Sets and Their Characteristic Functions

To avoid confusion, let me make a brief remark on the relation between talk about functions and talk about sets. In an earlier section I said that the semantic value of the expression 'smokes' is a set, while I said in the last section that its semantic value is a function. To say that the semantic value of the expression 'smokes' is a function is more in the spirit of type theory than to say that its semantic value is a set. But in many cases it is still preferable to talk about sets, since sets are intuitively more accessible than functions. This variation is harmless, since every set corresponds to a function, namely to the so-called *characteristic function* of the set. The characteristic function of a set is the function that maps any x contained in the set to the truth-value True and any x that is not contained in the set to the truth-value False. For instance, the characteristic function of the set of smoking entities is the function that maps any entity x to the truth-value True if x smokes and any entity x to the truth-value False if x does not smoke. Therefore, we can translate talk about the set of smoking entities into talk about the function that maps any entity x to the truth-value True if x smokes and any entity x to the truth-value False if x does not smoke. In an analogous way, we can translate talk about sets into talk about functions in other cases.[6] Note that although the talk about sets is more intuitive than the talk about functions, both ways to talk rely on rather technical notions. However, sets (or functions) have informal correlates. For instance, sets of entities (or functions from entities to truth-values) can informally be understood as properties of entities.[7] In the following we will see that other sets (or functions) have informal correlates too.

2.1.5 The Leading Principles of Type Theory

As already pointed out, the basic idea of type theory is that the meaning of a sentence results from the meaning of its parts via a process of functional application that is dictated by the syntactic structure of the sentence. In the following I consider extensional type theory, which explains how the truth-value of a sentence results from the semantic values of its parts in the described way. In extensional type theory possible semantic values are: (i) elements of D, the set of entities that exist in the world, (ii) elements of $\{1, 0\}$, the set of truth-values (where '1' stands for the truth-value True and '0' for the truth-value False) and (iii) functions, for instance functions from D to $\{1, 0\}$.[8] The three leading principles of extensional type theory are: (i) the semantic values of declarative sentences are elements of $\{1, 0\}$, (ii) the semantic values of referring expressions are elements of D and (iii) the semantic values of other expressions are functions that are assigned in such a way that the process of functional application, which is dictated by the syntactic structure of the sentence, results in a truth-value for the sentence.[9] The different kinds of semantic values are classified in *types* of semantic values. The semantic values of sentences and referring expressions are the basic types t (for truth-value) and e (for entity); other types are derived from the two basic types. In the following I first consider referring expressions in more detail. Then I explain how the three leading principles of extensional type theory help us to determine the semantic type of expressions other than declarative sentences or referring expressions.

6 Cp. Heim and Kratzer (1998: 24ff.) for details on sets and their characteristic functions.

7 However, this is only a very rough informal characterization of sets of entities, since a single set of entities can correspond to more than one property.

8 These are the possible semantic values in the framework developed by Heim and Kratzer (1998). There are other approaches according to which there are no expressions that have entities as semantic values (cp. Montague (1973)). These approaches will play a prominent role later in the book.

9 Cp. Forbes (2006: 16). The following presentation of type theory runs roughly along the lines of the one given by Forbes.

Referring Expressions

Paradigmatic examples of referring expressions are certain singular terms. Singular terms are proper names like ‘Paula’ or ‘John’ or indexical expressions like ‘I’ or ‘you’. To a first approximation, we can say that singular terms are expressions that pick out or that refer to a unique entity in the world. But this explication rules out that a proper name like ‘Sherlock Holmes’ is a singular term, since this name does not refer to an entity in the world.[10] To allow that such proper names are singular terms as well, we should modify the explication given above and rather say that singular terms are expressions that have the semantic function of picking out a unique entity in the world: they *purport* to pick out an entity in the word.[11] This explication leaves open the possibility that singular terms do not pick out a unique entity in the world and thus do not fulfill their semantic function. We can then reserve the term *referring expression* for singular terms that fulfill their semantic function and that have thus a semantic value of type e. Note that *referring* is a relation that only obtains between expressions that pick out a unique entity in the world and these entities, while *denoting* is a relation that obtains between arbitrary expressions and their semantic values. For instance, the name ‘Paula’ refers to Paula and denotes Paula, while the sentence ‘Paula smokes’ denotes the truth-value True but does not refer to this truth-value.

Intransitive Verbs

A sentence like ‘Paula smokes’ splits into the noun phrase ‘Paula’ and the verb phrase ‘smokes’, while the noun phrase ‘Paula’ consists in the noun ‘Paula’ and the verb phrase ‘smokes’ in the intransitive verb ‘smokes’.[12] This syntactic structure of the sentence can be represented with the aid of a syntactic tree as represented in figure 2.1 (‘S’ is short for sentence, ‘NP’ short for noun phrase, ‘N’ short for noun, ‘VP’ short for verb phrase and ‘IV’ short for intransitive verb). According to principle (i) of extensional type theory, the semantic value of the sentence is of type t and according to principle (ii), the semantic value of the noun ‘Paula’ is of type e. Therefore, we can already assign the types of semantic values specified in figure 2.2.[13] What is the semantic value of the intransitive verb ‘smokes’? According to principle (iii), the truth-value of a sentence results from a process of functional application. Thus, the semantic value of one of the expressions contained in the sentence must be a function. We already know that the semantic value of ‘Paula’ is not a function but an

[10] Some philosophers say that a proper name like ‘Sherlock Holmes’ refers as well, namely to an abstract fictional object (cp., e.g., Kripke (2013)). But even then, we should have a definition of singular term that allows proper names to be singular terms when they do not refer, as we should not preclude either that e.g. the non-fictional proper name ‘Vulcan’ is a singular term.

[11] Cp. Quine (1960: 96).

[12] Other than nouns, noun phrases are not part of the lexicon of a language. Noun phrases are items further up in the syntactic tree that are headed by a noun. *Mutatis mutandis* for other phrases.

[13] Note that nodes inherit the semantic values from their daughters, i.e. the semantic value of the noun phrase ‘Paula’ is simply the semantic value of the noun ‘Paula’.

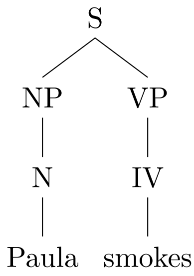

Figure 2.1

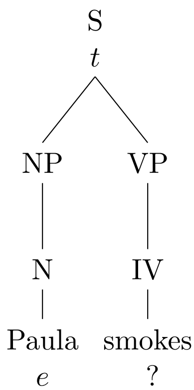

Figure 2.2

entity. Therefore, the semantic value of ‘smokes’ must be a function. What kind of function? The argument of the function is an entity and the value is a truth-value. Thus, it is a function from entities to truth-values (type $< e, t >$). More particularly, it is a function that maps any entity x to the truth-value True if x smokes and to the truth-value False if x does not smoke. We can thus complete the assignment of types as specified in figure 2.3:

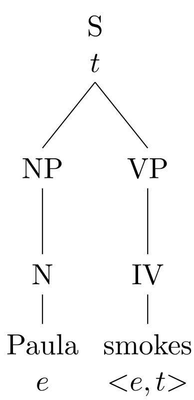

Figure 2.3

If the function denoted by ‘smokes’ is applied to Paula, then it yields the truth-value True and thus the sentence ‘Paula smokes’ obtains the truth-value True. Instead of saying that a function is applied to Paula and yields for Paula the truth-value True, we can informally say that a property is ascribed to Paula, namely the property of being a smoker. Since Paula is a smoker, the sentence is true.

Transitive Verbs

A sentence like ‘Paula loves John’ splits into the noun phrase ‘Paula’ and the verb phrase ‘loves John’.[14] The noun phrase ‘Paula’ consists in the noun ‘Paula’ and the verb phrase ‘loves John’ consists in the verb phrase ‘loves’ and the noun phrase ‘John’. The verb phrase ‘loves’ consists in the transitive verb ‘loves’ and the noun phrase ‘John’ in the noun ‘John’. We can represent the syntactic structure of the sentence as follows (‘TV’ is short for transitive verb):

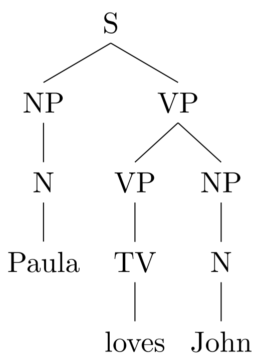

Figure 2.4

According to principle (i) of extensional type theory, the semantic value of the sentence is of type t and according to principle (ii), the semantic values of ‘Paula’ and ‘John’ are of type e. Therefore, we can already assign the semantic values specified in figure 2.5. What is the type of the semantic value of the transitive verb ‘loves’? To determine its type, we first need to determine the type of the semantic value of the verb phrase ‘loves John’. As we can see from the tree, the semantic value of the verb phrase ‘loves John’ combines with the semantic value of the noun phrase ‘Paula’ and yields the semantic value of the sentence ‘Paula loves John’. The semantic value of the noun phrase ‘Paula’ is of type e and the semantic value of the sentence ‘Paula loves John’ is of type t. Therefore, the semantic value of the verb phrase ‘loves John’ is a function from entities to truth-values (type $< e, t >$). More particularly, the semantic value of ‘loves John’ is a function that maps any entity x to the truth-value True if x

[14] Why does the sentence split into ‘Paula’ and ‘loves John’ and not, e.g., in ‘Paula loves’ and ‘John’? This is due to the fact that ‘loves John’ is a syntactic constituent, while ‘Paula loves’ is not. Cp. Adger (2003: 48ff.) for more on syntactic constituents.

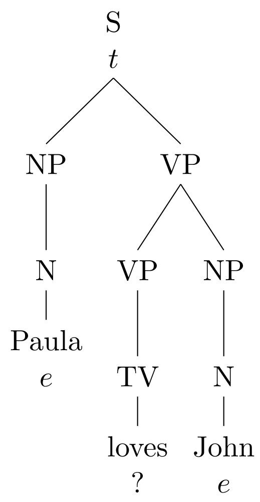

Figure 2.5

loves John and to the truth-value False if x does not love John. Thus, we can assign a further type of semantic value, as shown in figure 2.6:

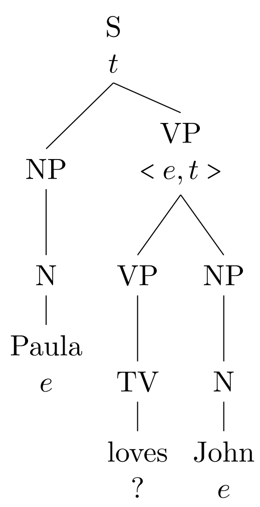

Figure 2.6

We can now determine the type of the semantic value of the transitive verb ‘loves’. As we can see from the tree, the semantic value of the verb phrase ‘loves’ combines with the semantic value of the noun phrase ‘John’ and yields the semantic value of the verb phrase ‘loves John’. The semantic value of the noun phrase ‘John’ is of type e and the semantic value of the resulting verb phrase ‘loves John’ is of type $< e,t >$. Therefore, the semantic value of the verb phrase ‘loves’ is a function from entities to functions of type $< e,t >$ (type $< e, < e,t >>$). More particularly, the semantic value of the verb phrase ‘loves’ is a function that maps any entity y to the function that maps any entity x to the truth-value True if x loves y and to the truth-value False if x does not love y. Thus, we can complete the assignment of semantic types as follows:

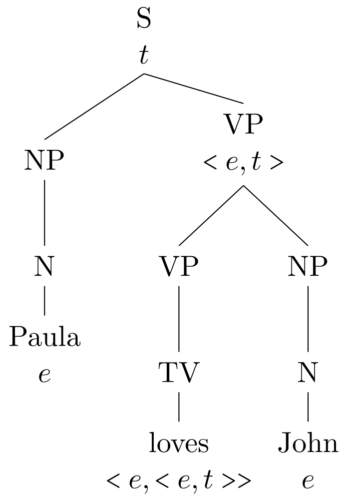

Figure 2.7

If the function denoted by the verb phrase 'loves' is applied to the entity denoted by the noun phrase 'John', then we obtain the function denoted by the verb phrase 'loves John'.[15] If the function denoted by the verb phrase 'loves John' is applied to the entity denoted by the noun phrase 'Paula', then we obtain the truth-value True if Paula loves John and the truth-value False if Paula does not love John. Instead of saying that the semantic value of a transitive verb like 'loves' is a function from entities to functions from entities to truth-values, we can informally say that the semantic value of 'loves' is a relation between entities. A sentence like 'Paula loves John' is true just in case Paula stands to John in the relation denoted by 'loves'.

Common Nouns and Adjectives

Common nouns and adjectives receive the same treatment. Let us take the sentence 'Paula is a woman' for illustration. This sentence splits into the noun phrase 'Paula' and the verb phrase 'is a woman'. The noun phrase 'Paula' consists in the noun 'Paula' and the verb phrase 'is a woman' consists in the copula 'is', the indefinite article 'a' and the common noun 'woman'. The simplified syntactic structure of the sentence can be represented as follows:

[15] Note that the function denoted by the verb phrase 'loves' cannot be applied to the entity denoted by the noun phrase 'Paula', since functional application is restricted to daughters of one and the same branching node and 'Paula' belongs to the NP node while 'loves' belongs to the VP node.